

# HYBRID ANALYTICAL-NUMERICAL METHODOLOGY FOR COMPUTATIONALLY EFFICIENT PRE-DESIGN ANALYSIS OF FLUID-STRUCTURE INTERACTION

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**Abstract.** Analytical approach to modeling the interaction between submerged elastic structures and non-stationary loads has long been recognized as an attractive tool of engineering analysis, especially at the pre-design stage where it has been particularly valued for its high computational efficiency. At the same time, the approach has a number of limitations, the most regrettable one being its inability to handle geometries that are more complex than the basic ones such as a spherical or cylindrical geometry. We present an attempt to overcome this limitation while still preserving the much favored computational efficiency by introducing a hybrid methodology that combines the analytical and finite-element approaches. We then validate the methodology using available experimental data and show that a good agreement with the experiments is observed.

## 1 INTRODUCTION

In recent years, an efficient semi-analytical methodology has been developed for modelling the interaction between shell structures and non-stationary loads, e.g. [1-4]. The methodology was based on the use of the so-called “response functions”, the functions that only depend on the geometry of the system and not on the properties of the fluid(s) and structure(s). The methodology was extensively validated and has proven to be an attractive choice for the use by the practitioner due to its high computational efficiency; it is particularly attractive when an extensive parametric analysis of the system is intended, as often is the case at the pre-design stage.

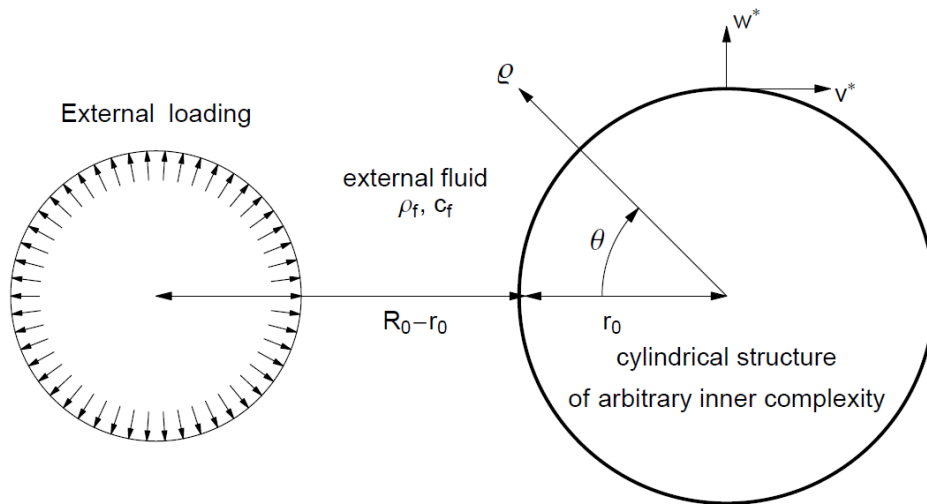
Along with its significant advantages, the methodology also has some rather serious limitations. In particular, in its present form it is only applicable to very specific geometries of the structure such as cylindrical and spherical. This feature significantly limits the applicability of the methodology due to the fact that most industrial systems possess higher geometrical and/or material complexity. In an attempt to overcome this limitation, we propose a hybrid model where the structural part is handled using FEM, while the fluid domain is

modelled using the response-functions-based methodology.

We show that such an approach results in a model that is capable of accurately simulating the shock response of many commonly encountered in engineering practice structures that have a degree of geometrical and/or material complexity, while being more computationally efficient than “pure-FEM” approaches. Thus, the proposed methodology is demonstrated to combine the versatility of FEM with the computational efficiency of the response-functions-based methodology.

## 2 MATHEMATICAL MODEL

We consider a structure of the cylindrical outer shape, of radius  $r_0$ , and of arbitrary inner structural complexity submerged into inviscid, irrotational, and linearly compressible fluid with density  $\rho_f$  and sound speed  $c_f$ . The transverse and normal displacements of the outer surface of the structure are  $v^*$  and  $w^*$ , respectively. The geometry of the problem is shown in Figure 1.



**Figure 1:** Geometry of the problem

We assume that the structure is subjected to a high-frequency, non-stationary external loading.

The fluid domain is modeled using the wave equation, an assumption which, combined with the simplicity of the geometry of the interface, enables the use of the classical apparatus of the mathematical physics. For the structural domain, however, we do not make any simplifying assumptions regarding its inner geometry, and model it assuming that the structure’s material is linearly elastic and isotropic.

For the fluid domain, therefore, we obtain solution using the approach that we have developed in our earlier work [1-4], that is, the Laplace transform with respect to time is first applied to the wave equation, and then the separation of the spatial variables is used to yield the general solution in the form of a linear combination of two modified Bessel functions. Upon applying the boundary conditions and inverting the resulting expressions, the pressure

components are obtained in the form of Fourier series with time-dependent coefficients. For the radiated pressure (the component that plays the key role in the proposed methodology), these coefficients are (the dimensionless form of the normal displacement is used, thus the asterisk is not present):

$$p_n^r = - \int_0^t \frac{d^2 w_n(\eta)}{d\eta^2} \xi_n^e(r, t - \eta) d\eta \quad (1)$$

where  $\xi_n^e(r, t)$  are the response functions of the problem with the Laplace transforms given by

$$\Xi_n^e(r, s) = - \frac{K_n(rs)}{sK_n'(s)} \quad (2)$$

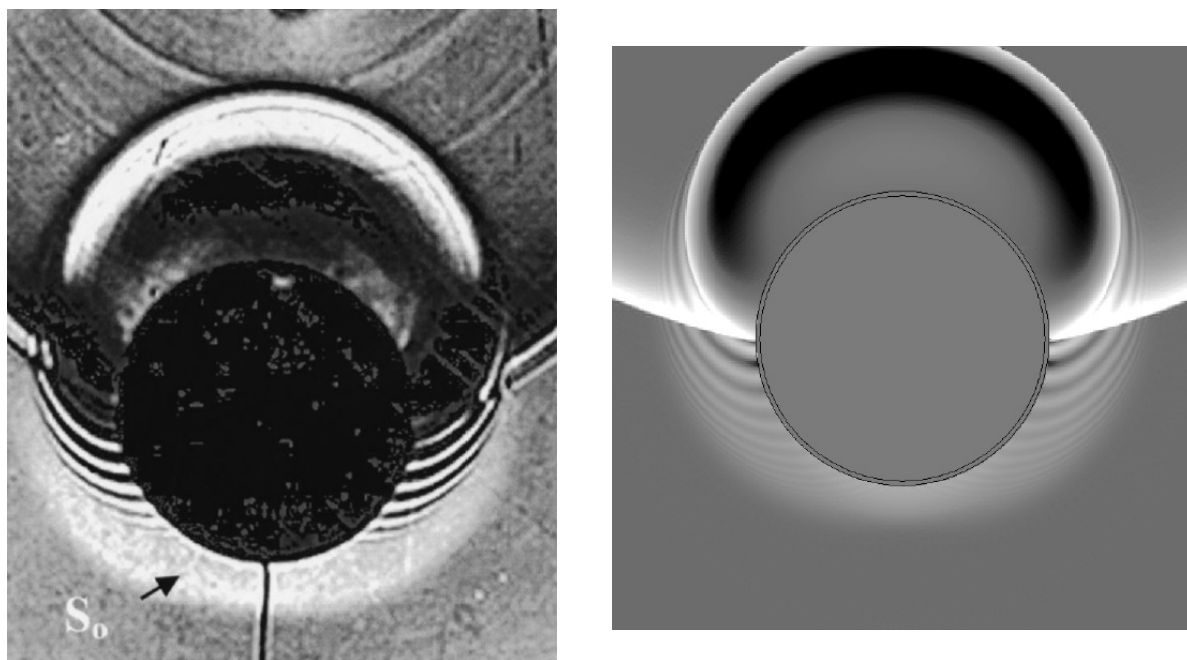
where  $K_n$  is the modified Bessel function of the second kind of order  $n$ , and  $w_n$  is the time-dependent component of the  $n$ -th harmonic of  $w$ . All the details of the methodology can be found in our earlier work [1-4].

For the structural part, the standard finite-element approach has been adopted, and we do not include any of the details here. The fluid and the structural parts are coupled at each time step, with the latter receiving the hydrodynamic loading from the former and passing back the normal displacements of the structural surface; the process is accompanied by the constant switching between the functional and modal forms of the quantities concerned.

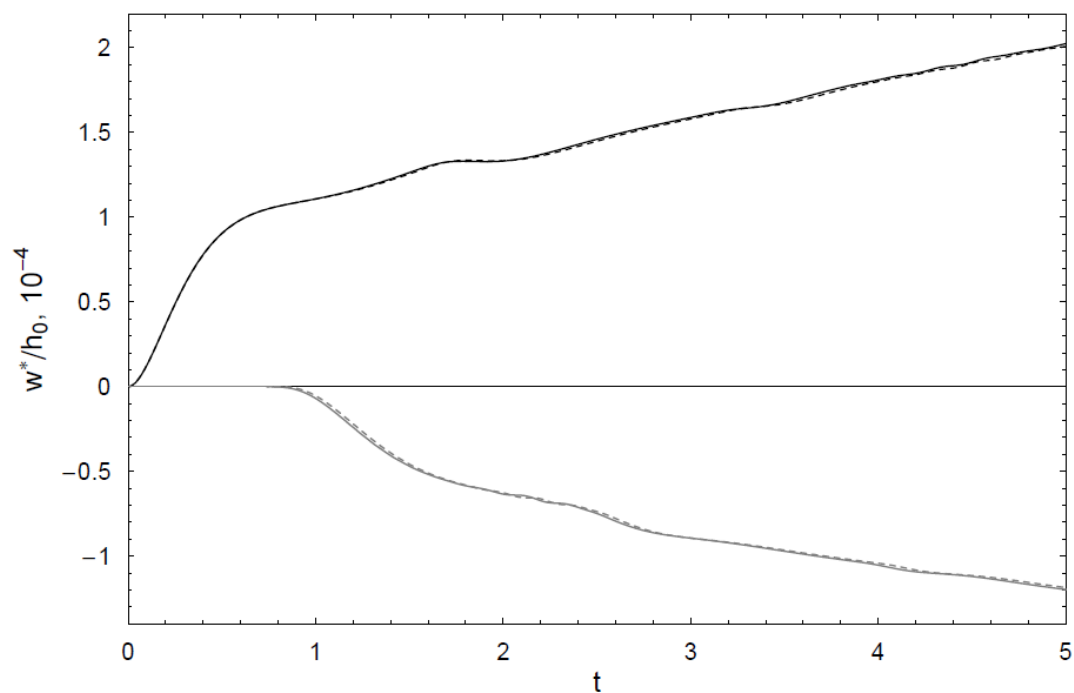
### 3 RESULTS AND DISCUSSION

The ultimate objective of this study is to be able to simulate the interaction with structures that have a certain degree of geometrical complexity. But prior to attempting such simulations, we needed to validate the developed methodology. To that end, we considered a typical steel shell with the thickness-to-radius ratio of 0.03 submerged in water and subjected to a point-source acoustic pulse, and carried out two comparisons - one with the results of an experimental study [5], Figure 2, and the other with the results produced by the simulations based on a semi-analytical model [4], Figure 3, the former aimed at establishing that the methodology adequately reproduces the structure of the hydrodynamic field induced during the interaction, and the latter ensuring that the structural dynamics is also accurately represented. In both cases, a very good agreement has been observed, thus providing a rather solid evidence in favor of the developed approach. We note that the fact that the good match was seen for the two rather different aspects of the interaction is quite comforting as far as the use of the approach for modeling more complex systems is concerned.

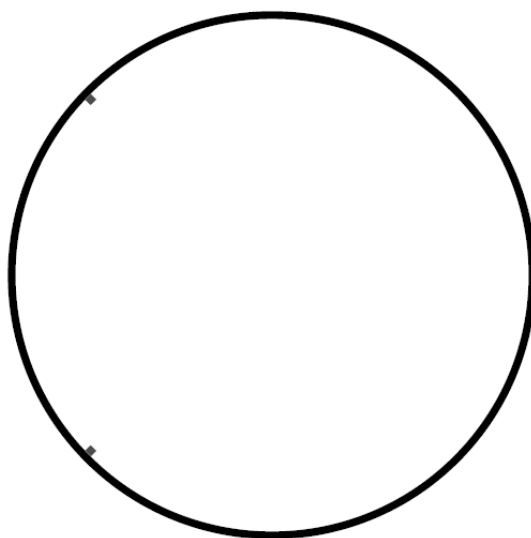
We then considered a more structurally complex system, namely, a steel shell of radius 1 m and thickness 0.03 m, with two attached masses positioned at  $\pm 45^\circ$ , of the radial extension of 0.03 m and angular one of  $1.8^\circ$ , also made of steel; the aim of choosing such relatively “light” masses was to demonstrate that even a minor structural modification produces rather significant differences as far as the radiated hydrodynamic pattern is concerned. Figure 5 shows the snapshots of the radiated hydrodynamic field during the early interaction for the shell with and without the attached masses. The effect of the attached masses is apparent.



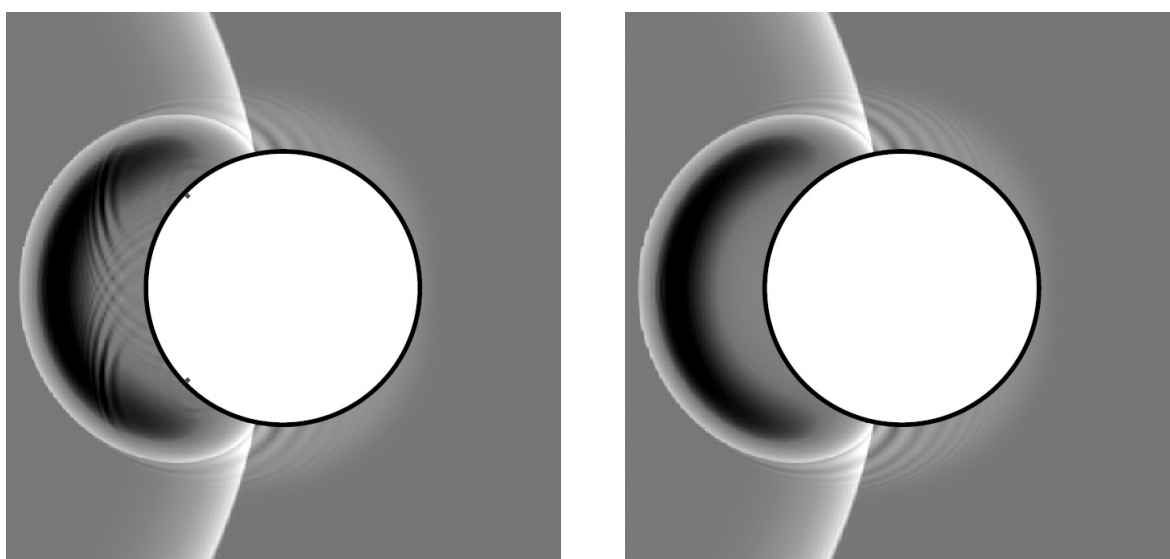
**Figure 2:** Numerically simulated hydrodynamic field around a steel shell with  $h_0/r_0=0.03$ , right, compared with an experimental image, left [reprinted with permission from Derbesse, L., Pernod, P., Latard, V., Merlen, A., Decultot, D., Touraine, N., and Maze, G. 2000 Acoustic scattering from complex elastic shells: visualization of  $S_0$ ,  $A_0$  and  $A$  waves. Ultrasonics 38, 860-863, Figure 1 (b), © 2000 Elsevier].



**Figure 3:** Normal displacements of a steel shell with  $h_0/r_0=0.03$  computed using a semi-analytical approach based on the Reissner-Mindlin model (dashed lines) and using the present approach (solid lines) evaluated at the head point (black lines) and the tail point (grey lines).

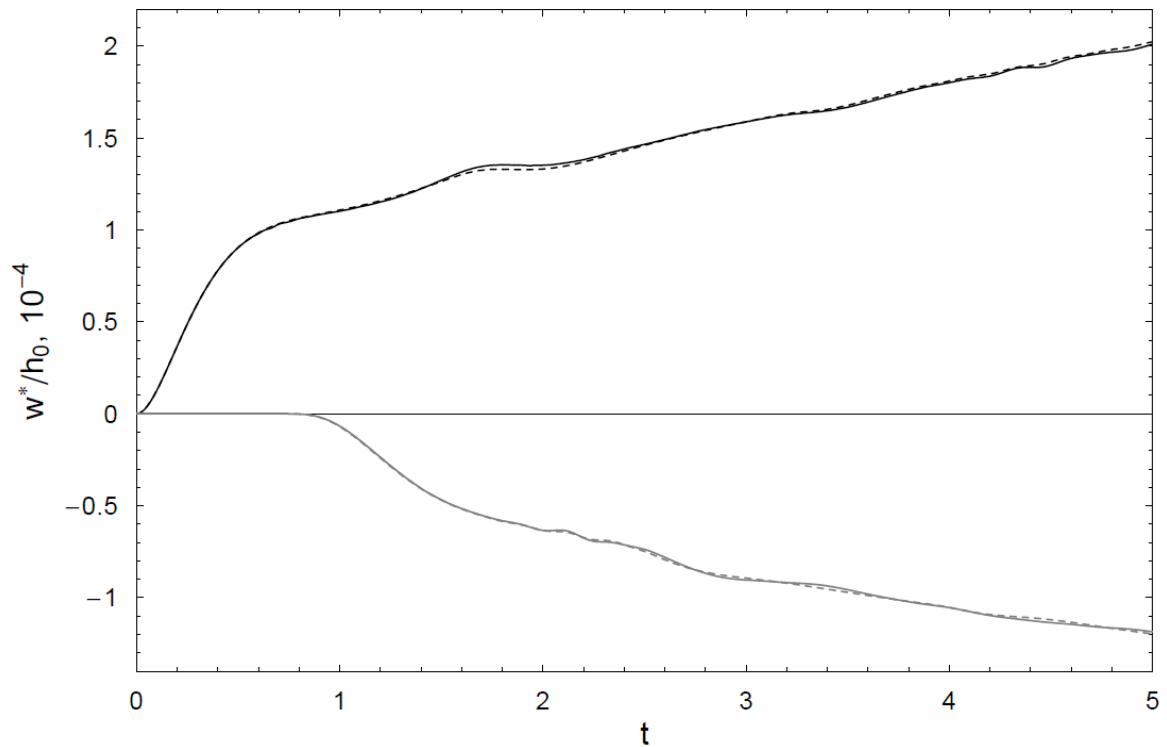


**Figure 4:** Shell with attached masses.



**Figure 5:** Numerically simulated hydrodynamic field around a steel shell with  $h_0/r_0=0.03$  and two small attached masses during the early interaction (left), and around the same shell at the same instant but without attached masses (right).

We point out that although the hydrodynamic patterns are noticeably different, the structural dynamics is hardly affected at all, as is evident from the time-histories of the normal displacements, Figure 6. This observation highlights the importance of the focused analysis of the hydrodynamic fields carried out along with the structural analysis when a complete understanding of all aspects of the interaction is necessary.



**Figure 6:** Normal displacements of a steel shell with  $h_0/r_0=0.03$  and no structural enhancements (dashed lines) compared to the normal displacements of the same shell with attached masses (solid lines) evaluated at the head point (black lines) and the tail point (grey lines).

#### 4 CONCLUSIONS

We have proposed a hybrid analytical-numerical methodology for simulating the interaction between a structure which has a cylindrical outer shape but arbitrary inner complexity. The methodology combines the analytical approach based on the classical apparatus of mathematical physics for the fluid domain with the finite-element approach for the structural domain, with coupling occurring on the interface at each time step.

The methodology has been validated using both available semi-analytical solution and experimental data, and a good agreement was observed in both cases.

We then considered a more complex system, namely, a cylindrical shell with two attached masses, and demonstrated the capabilities of the model by simulating both the fluid and structural dynamics of such a system. In particular, we showed that although the structural effect of the attached masses, as manifested in the time-histories of the normal displacements, is very insignificant, their effect on the overall hydrodynamic pattern is quite pronounced, thus highlighting the necessity of a careful consideration of all aspects of the interaction.

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